Lecture 2: Basic Graph Algorithms

CS 539 / ECE 526

Distributed Algorithms

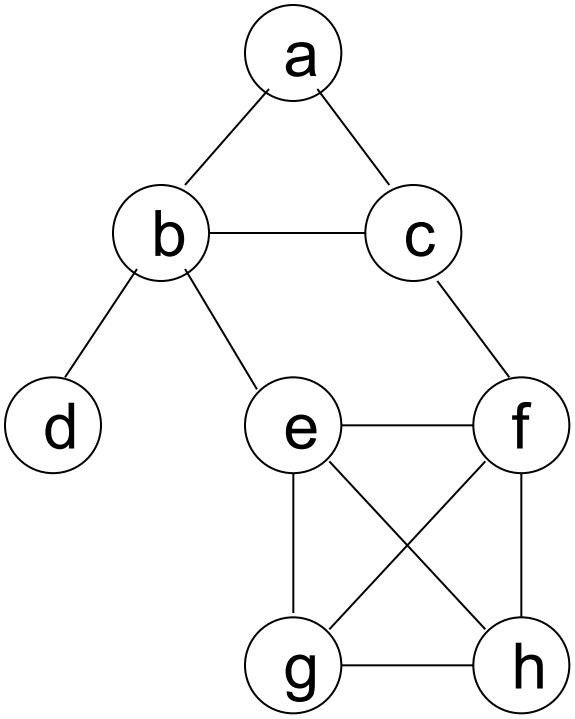
Some slides borrowed from Jennifer Welch's class CSCE 668 at Texas A&M

Outline

- Simple broadcast using flooding
- Broadcast & convergecast assuming a spanning tree
- Find a spanning tree
- Breadth-first search

The Broadcast Problem

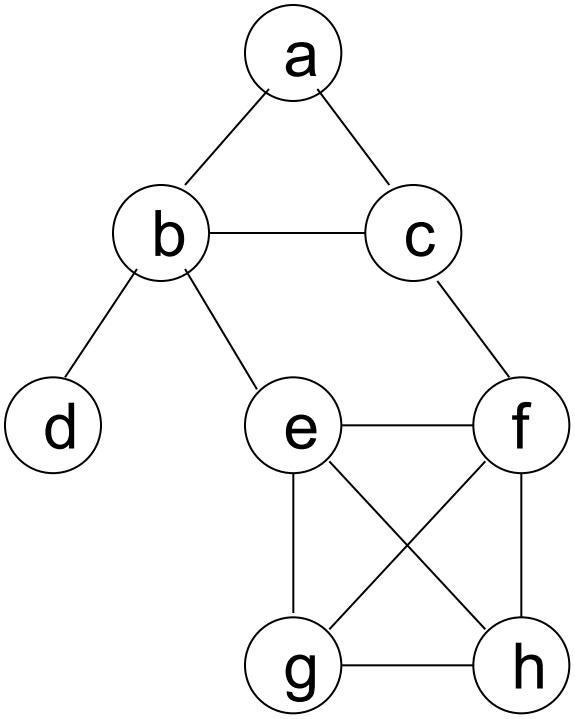
- Distributed processes / nodes
- One node has a piece of information M
- Want to send M to all nodes
- Assumption today:
  - Message passing
  - Generic graph that is connected
  - No link or node faults
  - Sync or async



Flooding Broadcast (Async)

- Initially, broadcaster sends M to all neighbors
- Upon receiving M **for the first time**

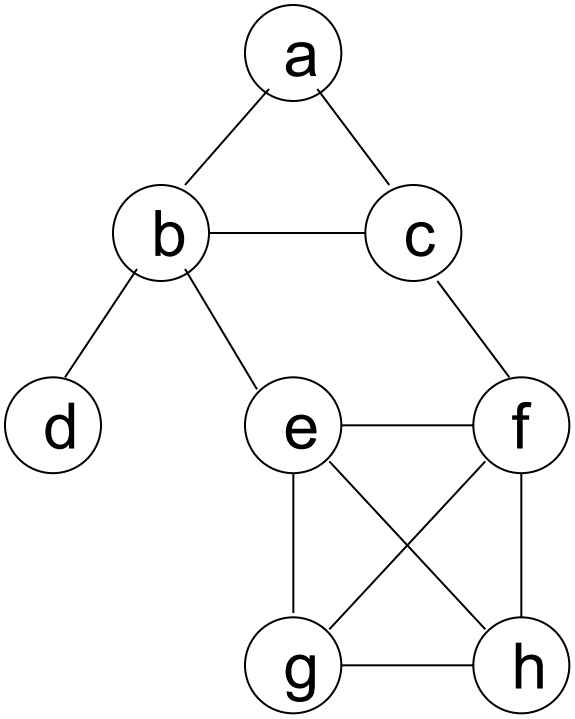
  Send to all neighbors



Flooding Broadcast (Lockstep Sync)

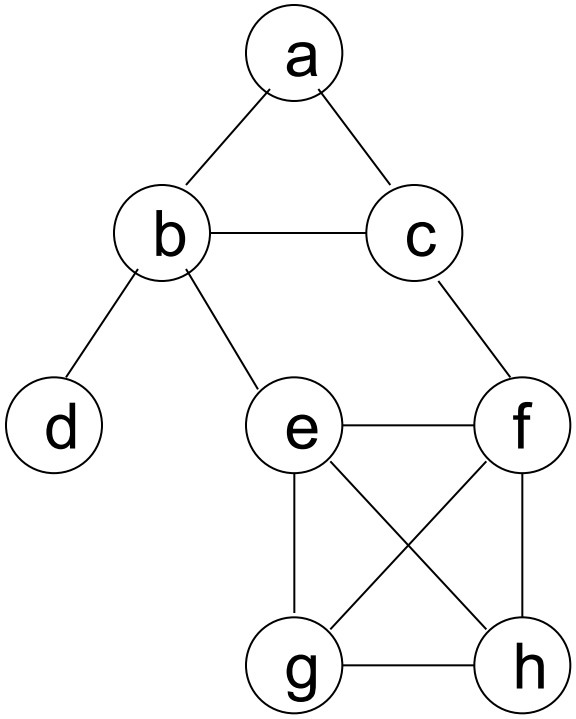
- Round 1: broadcaster sends M to all neighbors
- Round r > 1: if receiving M **for the first time** in the last round

  Send to all neighbors



Flooding Broadcast Correctness

- Graph connected → exist a path from broadcaster → induction based on distance
- "Obvious”

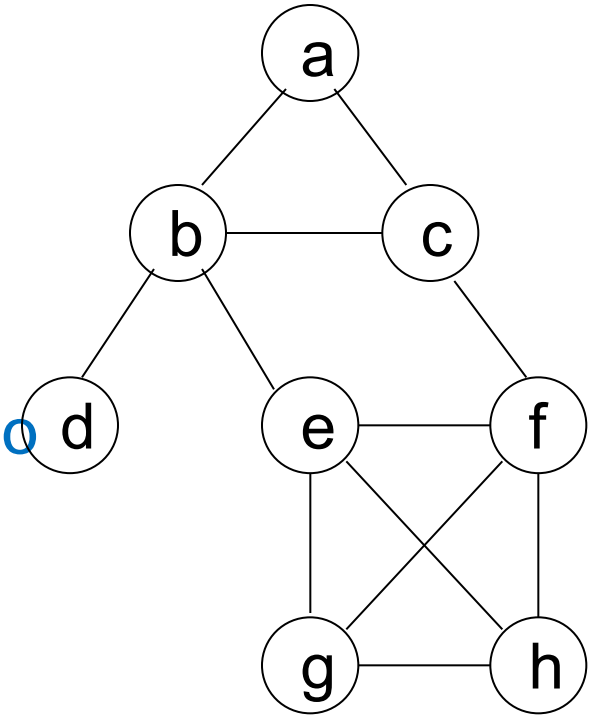


Recall Efficiency Metrics

- Time complexity
  - For synchrony: # of rounds (aka round complexity)
  - Can be extended to asynchrony
- Communication complexity
  - Can be measured in # of msgs or bits
- Computation and space complexity
  - Not too different from non-distributed
  - Often ignored, will discuss when important

Flooding Broadcast Efficiency

- Round complexity:
  - D (graph diameter)
- Communication complexity:
  - 2|E| msgs (E is the set of edges)
    - 1 msg per direction per edge
  - Do not send M back to those who sent M to me?
    - Useful in some cases, but not always, e.g., complete graph



Outline

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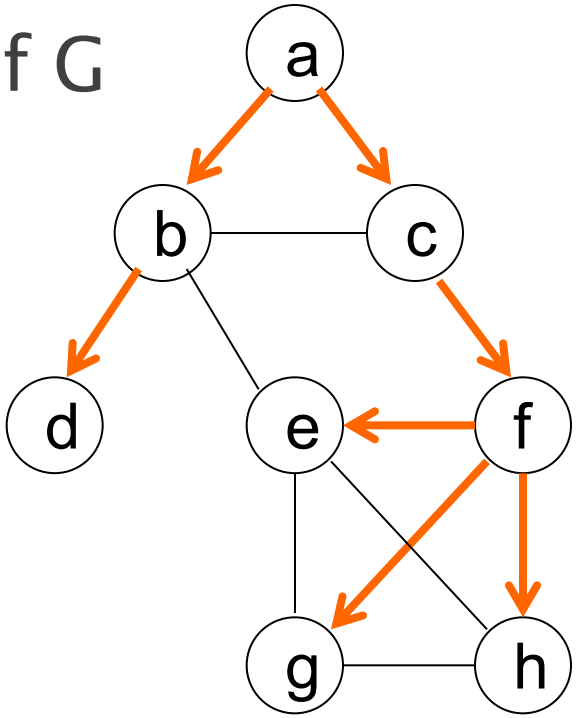
- Broadcast & convergecast assuming a spanning tree
- Find a spanning tree
- Breadth-first search

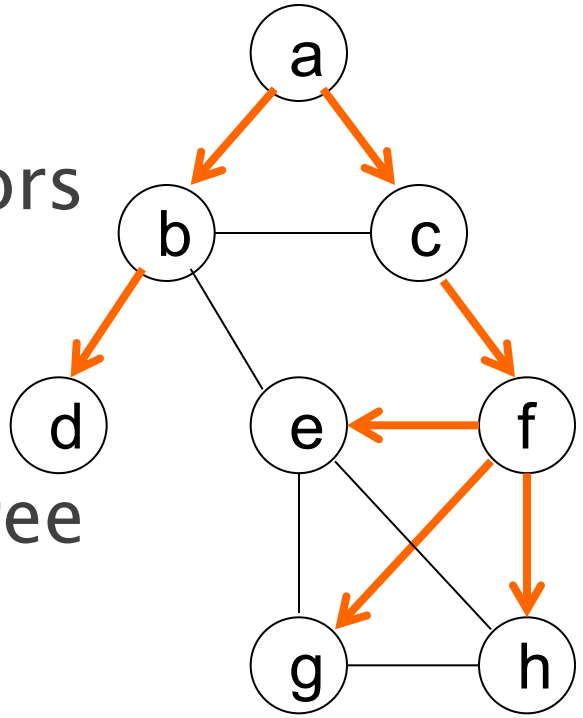
Spanning Tree

- Let G = (V, E) be a connected undirected graph
- T = (V, E') is a spanning tree of G
  - E' is a subset of E
  - T is connected
  - T has no cycles



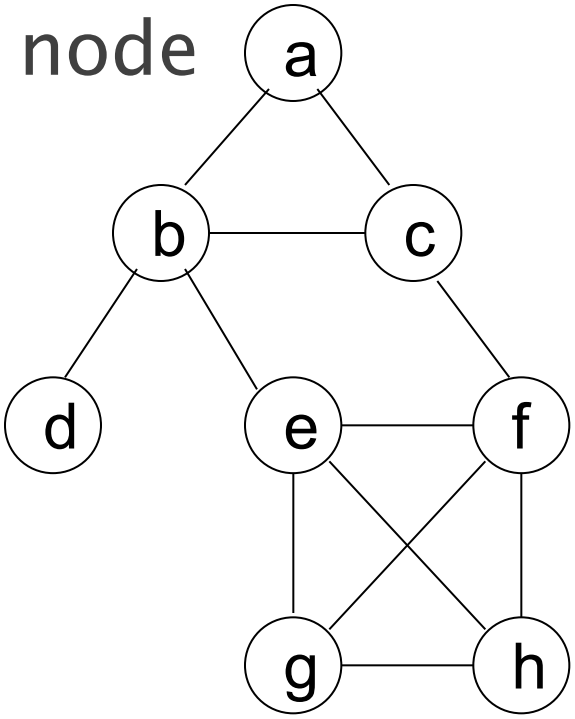
Broadcast using a Spanning Tree

- Assume a spanning tree has been build, i.e., each node maintains parent & list of children
- Similar to flooding, send to all children instead of all neighbors
- Correctness: same
- Round complexity: depth of tree
- Msg complexity: |V| - 1
  - No longer has the factor 2



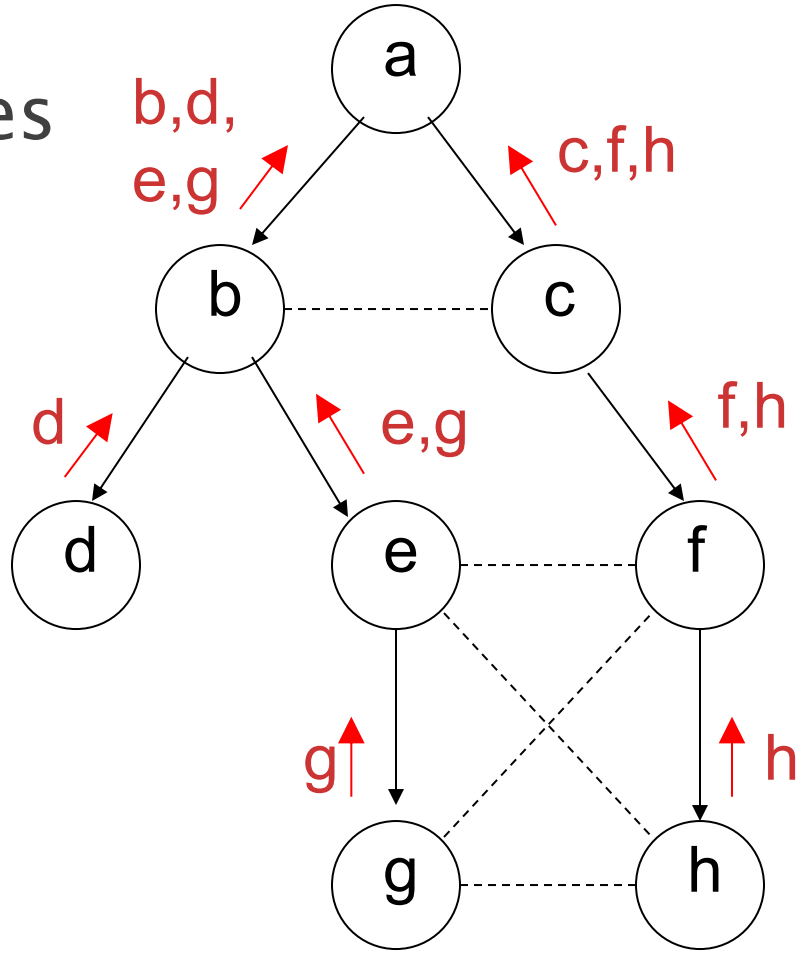
The Convergecast Problem

- Distributed processes / nodes
- Each node has a piece of information
- Want to **collect** all info at one node
  - Variants: sum, max, …
- Assumption today: same



Convergecast via a Spanning Tree

- Dotted lines: non-tree edges
- Solid arrows: tree edges



Convergecast via a Spanning Tree

- Initially, each leaf node sends input to parent
- Upon receiving from all children

  Send to parent $f(m_1, m_2, \ldots, m_c)$

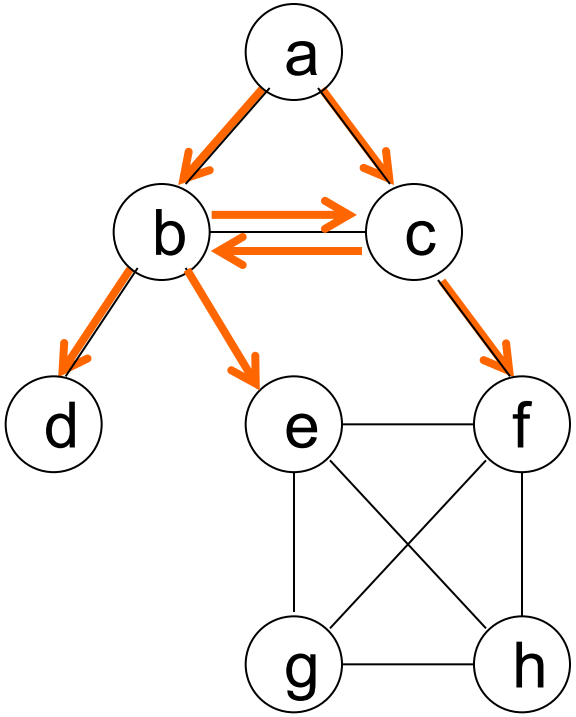
  (Showing async version here, sync version similar)
- Correctness: similar induction
- Efficiency: D rounds, |V| - 1 msgs
  - Bits depend on shape of spanning tree in general
  - (|V|-1)|M| for functions such as sum and max

Outline

- Simple broadcast using flooding
- Broadcast & convergecast assuming a spanning tree
- Find a spanning tree from specified root
- Breadth-first search

Spanning Tree from Specified Root

- Can augment the flooding algorithm
  - Record parent
  - Reply to parent so parent can record children



Spanning Tree via Flooding (Sync)

- Round 1: root sends recruit msg to all neighbors
- Round r > 1:

    If receiving a recruit msg in round r-1

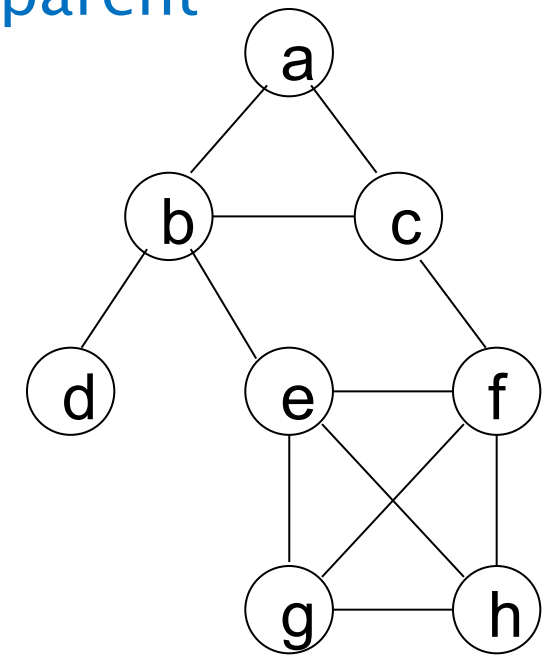
        If this is first time **(ties broken arbitrarily)**

            record parent & send yes to parent

            send recruit to all neighbors

    If receiving yes from node j

        Add j as a child



Spanning Tree via Flooding (Async)

- Initially: root sends recruit msg to all neighbors
- **Upon** receiving a recruit msg

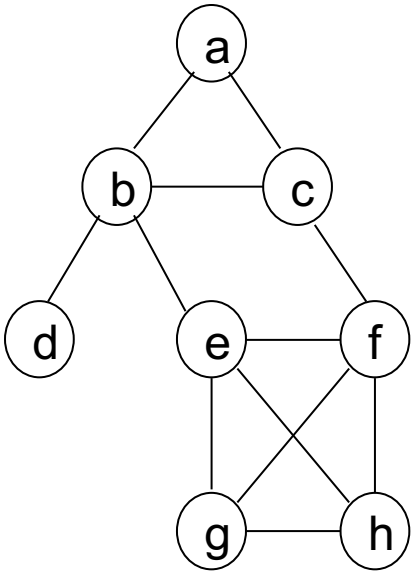
  If this is first time **(ties never occur in async)**

  record parent & send yes to parent

  send recruit to all neighbors

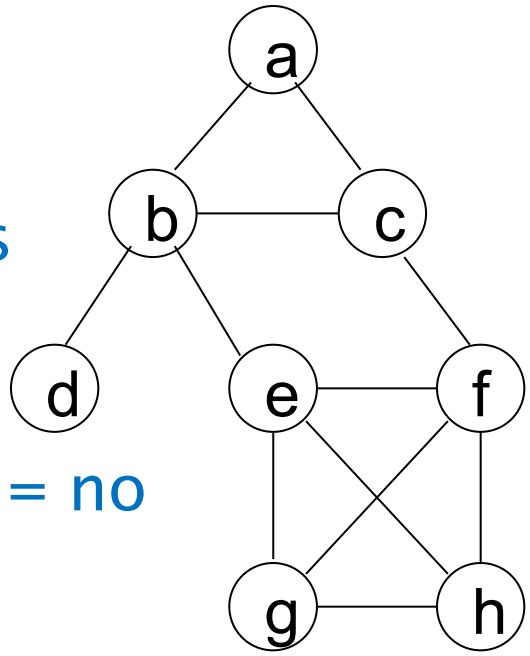
Upon receiving yes from node j

Add j as a child



Spanning Tree from Specified Root

- Can augment the flooding algorithm
  - Record parent
  - Reply to parent so parent can record children
- When do nodes terminate?
  - Reply to non-parent so that it does not wait forever
  - Not a problem in sync, lack of yes = no (observation from student)



Spanning Tree via Flooding (Async)

- Initially: root sends recruit msg to all neighbors
- **Upon** receiving a recruit msg

    If this is first time **(ties never occur in async)**

        record parent & send yes to parent

        send recruit to all neighbors

    Else:

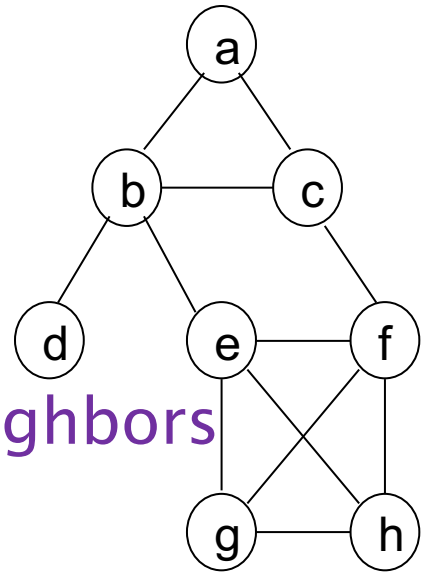
        reply with no

Upon receiving yes from node j

    Add j as a child

Upon receiving yes or no from all neighbors

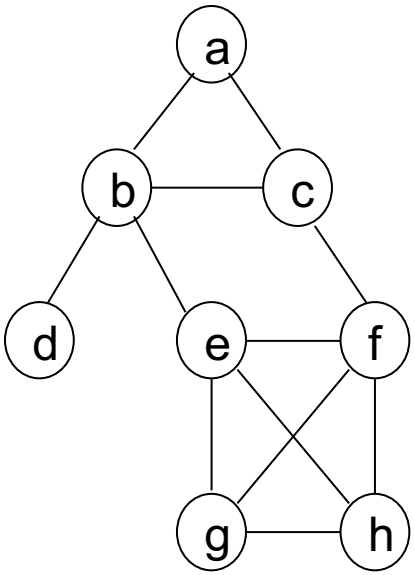
    Terminate



Spanning Tree via Flooding (Async Detailed)

Initially: parent = NULL, children = {}, pending = neighbors;
    for each j in neighbors
        root sends “recruit” to j

Upon receiving “recruit” from j
    If parent == NULL
        parent = j
        Send “yes” to j
        for each j in neighbors:
            Send “recruit” to j
    Else: send “no” to j
Upon receiving “yes” from j
    children = children U {j}
    pending = pending \ {j}
Upon receiving “no”
    pending = pending \ {j}
Upon pending = {}
    Terminate

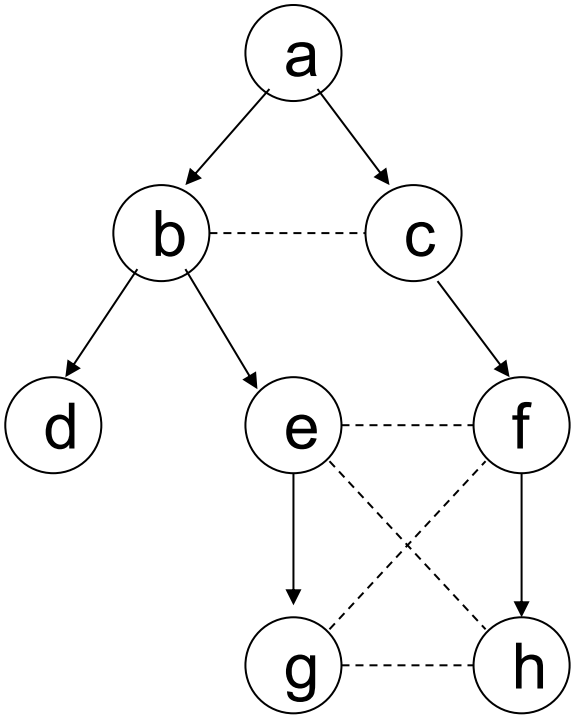


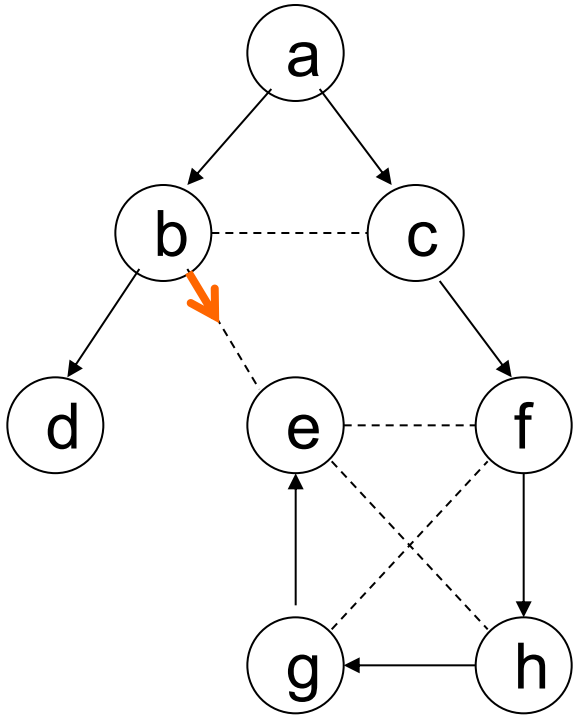
Spanning Tree via Flooding

- Correctness:
  - Every node has at most one parent
  - Every node has a parent
    - Graph connected, induction on distance
  - Node i considers j parent if and only if j considers i child
- Efficiency: same as flooding broadcast
  - Round: Diameter of graph (+1)
  - Communication: 2|E| msgs, O(|E|) bits

Spanning Tree via Flooding

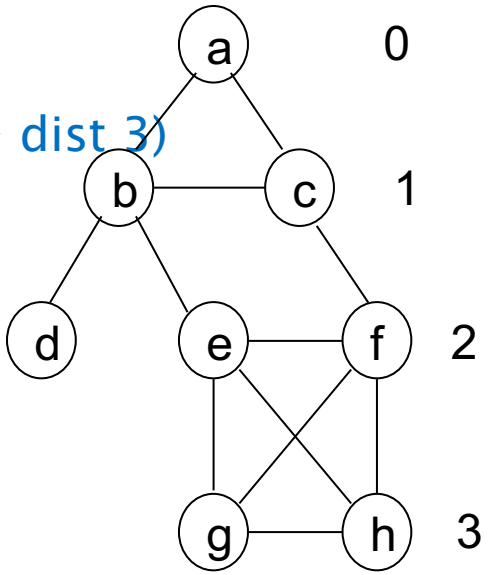
- Extra nice property under sync: breadth-first search
- Does not hold under async





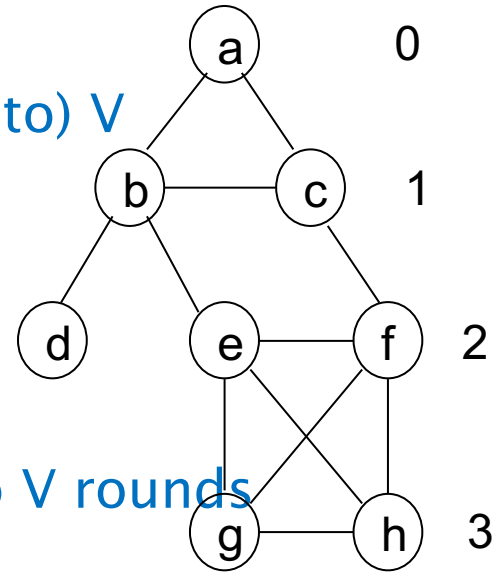
BFS in Async

- One idea: synchronized (yes, in async) "wavefronts"
  - Root recruit distance-1 nodes (root's neighbors)
  - Wait for confirmation from all of these
  - Recruit distance-2 nodes (root → dist-1 → dist-2)
  - Convergecast confirmations back to root
  - Recruit distance-3 nodes (root → dist-1 → dist-2 → dist-3)
  - Convergecast confirmations back to root
  - …



BFS in Async Efficiency

- One idea: synchronized (yes, in async) "wavefronts"
- Comm complexity:
  - O(|E| + |V|*D)
  - Each edge sees recruit once and yes/no once
  - D broadcast / convergecast each costing (up to) V
- Round complexity:
  - $O(D^2)$, D broadcast / convergecast each up to V rounds
    - More accurately: 1 + 2 + 3 + … + D



Summary

- Broadcast & convergecast via a spanning tree (sync and async)
- Find a spanning tree from specified root using flooding (sync and async)
- Breadth-first search
  - Easy in sync, need synchronization in async