UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Lecture 9-10: Fault Bounds of Consensus

CS 539 / ECE 526

Distributed Algorithms

Today: Fault Bounds

- How many faults can we tolerate?
- Highly sensitive to various conditions
- All the fault bounds in this lecture are **tight**

Outline

- Fault bounds in synchrony
  - Byzantine agreement
  - Byzantine without signatures
  - Total-order broadcast and Replication
- Fault bounds in asynchrony
  - Broadcast
  - All other problems (FLP impossibility)
- Partial synchrony
  - Crash
  - Byzantine

Fault Bounds So Far

- Synchronous crash broadcast: $f < n$ (flooding)
- Synchronous Byzantine broadcast with signatures: $f < n$ [Dolev-Strong, 1983]

- How about Agreement?
- How about without signature?
- How about asynchrony?

Recall Agreement

- n parties, each has an input x_i , up to f faulty

- Safety: no different outputs
- Liveness: everyone outputs
- Validity: every honest inputs x → every honest outputs x

Recall Agreement Validity

- Every honest inputs x → every honest outputs x
- Some examples: what should the output be given following inputs?
  - Binary inputs: 1, 1, 1, 1, 1?
    - Must be 1
  - Binary inputs: 0, 1, 1, 0, 1?
    - Must be 1 if both 0s are Byzantine inputs
    - Otherwise, either 0 or 1
  - Multi-value inputs: 3, 3, 5, 2, 3, 3, 3?
    - Must be 3 if 5 and 2 are Byzantine inputs
    - Otherwise, anything is fine

Recall Agreement Validity

- Every honest inputs x → every honest outputs x
  - Not meant to be useful
  - Just an easy condition to rule out trivial solutions

- Why don't we define a more useful validity?
- Turns out it may make the problem too hard (problem set 2)

Broadcast to Agreement

- Byzantine broadcast (BB) gives BA if $f < n/2$
  - Every party invokes BB on its input
  - Every party gets an agreed upon vector
    - Byzantine → Any value in that position of vector
  - Everyone picks the most frequent value
    - $f < n/2$ needed for validity of Byzantine agreement

Broadcast to Agreement

- Safety: same vector, same way to pick
- Liveness: obvious
- Validity: if all honest have same input x, then x will be the most frequent (since $f < n/2$)

- Round complexity: same as BB
- Communication complexity: *n* times BB

Byzantine Agreement Fault Bound

- Byzantine agreement is not solvable if $f \geq n/2$
  - Proof: Divide parties into two groups P and Q such that $|P| \leq f$ and $|Q| \leq f$
  - Scenario I: P are honest and receive input v; Q are Byzantine and behave as if they receive input v'
    - P commits v due to validity

Byzantine Agreement Fault Bound

- Byzantine agreement is not solvable if $f \geq n/2$
  - Proof: Two groups $|P| \leq f$ and $|Q| \leq f$
  - Scenario I: P honest & receive v, Q Byzantine & receive v' → P commit v due to validity
  - Scenario II: Q honest & receive v', P Byzantine & receive v → Q commit v' due to validity
  - Scenario III: P receive v, Q receive v', both honest
    - P cannot distinguish III from I & commit v
    - Q cannot distinguish III from II & commit v'

Broadcast to Agreement

- Crash tolerant agreement for $f < n$ with a modification to validity
  - Every party invokes broadcast on its input
  - Every party gets an agreed upon vector
    - Crash → possibly $\perp$ in that position of vector
  - Everyone picks the most frequent non-$\perp$ value

Broadcast to Agreement

- Problem with standard validity when $f \geq n/2$
  - Example: inputs 0, 0, 0, 1, 1, 1. How to pick in a tie?
  - Pick 0? What if all three parties with input 0 crash right before they output?
    - All three non-faulty have input 1, must output 1
    - Symmetric problem for picking 1
- Modified validity: if all n parties input x, all non-faulty parties output x

Broadcast to Agreement

- Safety: same vector, same way to pick
- Liveness: obvious
- Validity: all n parties input x → agreed-upon vector has only x and ⊥ → all pick x (non-⊥)

- Round complexity: same as broadcast
- Communication complexity: *n* times broadcast

Fault Bound without Signatures

- BA or BB without signatures: f < n/3

  [Lamport-Shostak-Pease, 1982]

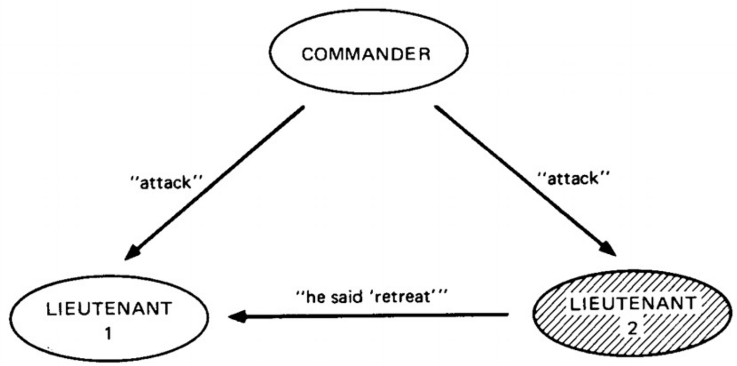


Fig. 1. Lieutenant 2 a traitor.

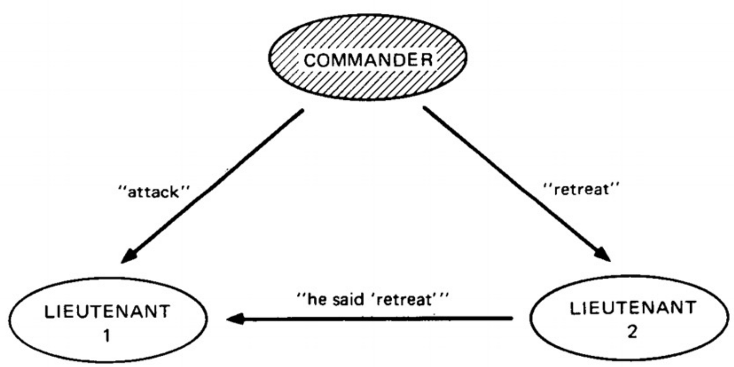


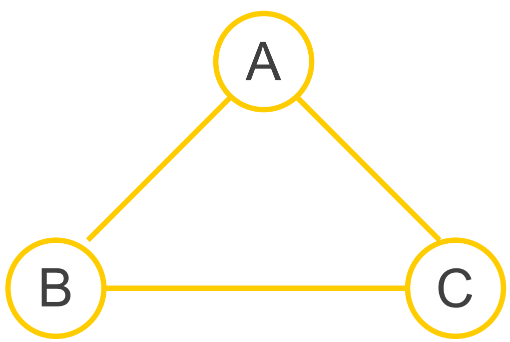
Fig. 2. The commander a traitor.

Fault Bound without Signatures

- BA or BB without signatures: $f < n/3$
- Previous argument was handwavy
  - We are trying to prove *No* algorithm works
  - Cannot assume how the protocol works
- Rigorous proof next [Fischer-Lynch-Merritt, 1986]
  - Step 1: no BA solution for $n = 3$, $f = 1$
  - Step 2: generalize to any $n \leq 3f$

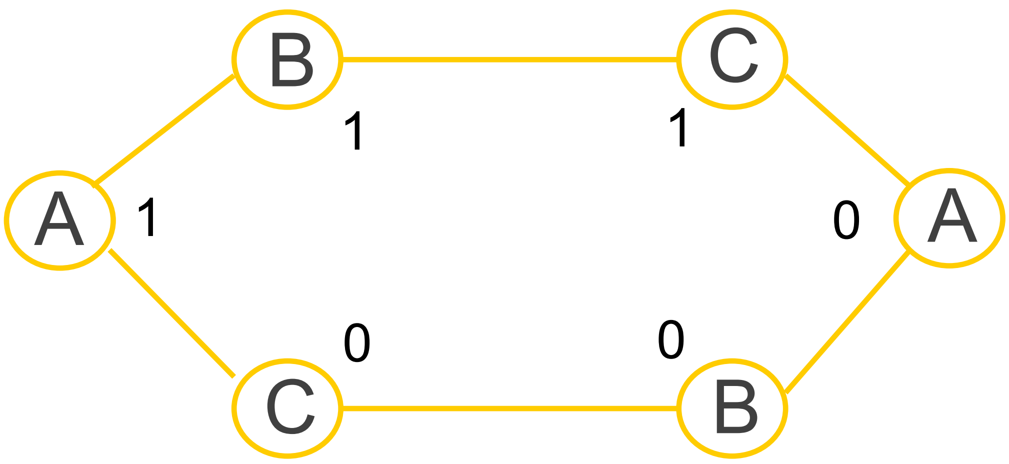
Fischer-Lynch-Merritt Proof

- Suppose for contradiction that there exists an algorithm that solves BA with $n = 3$, $f = 1$

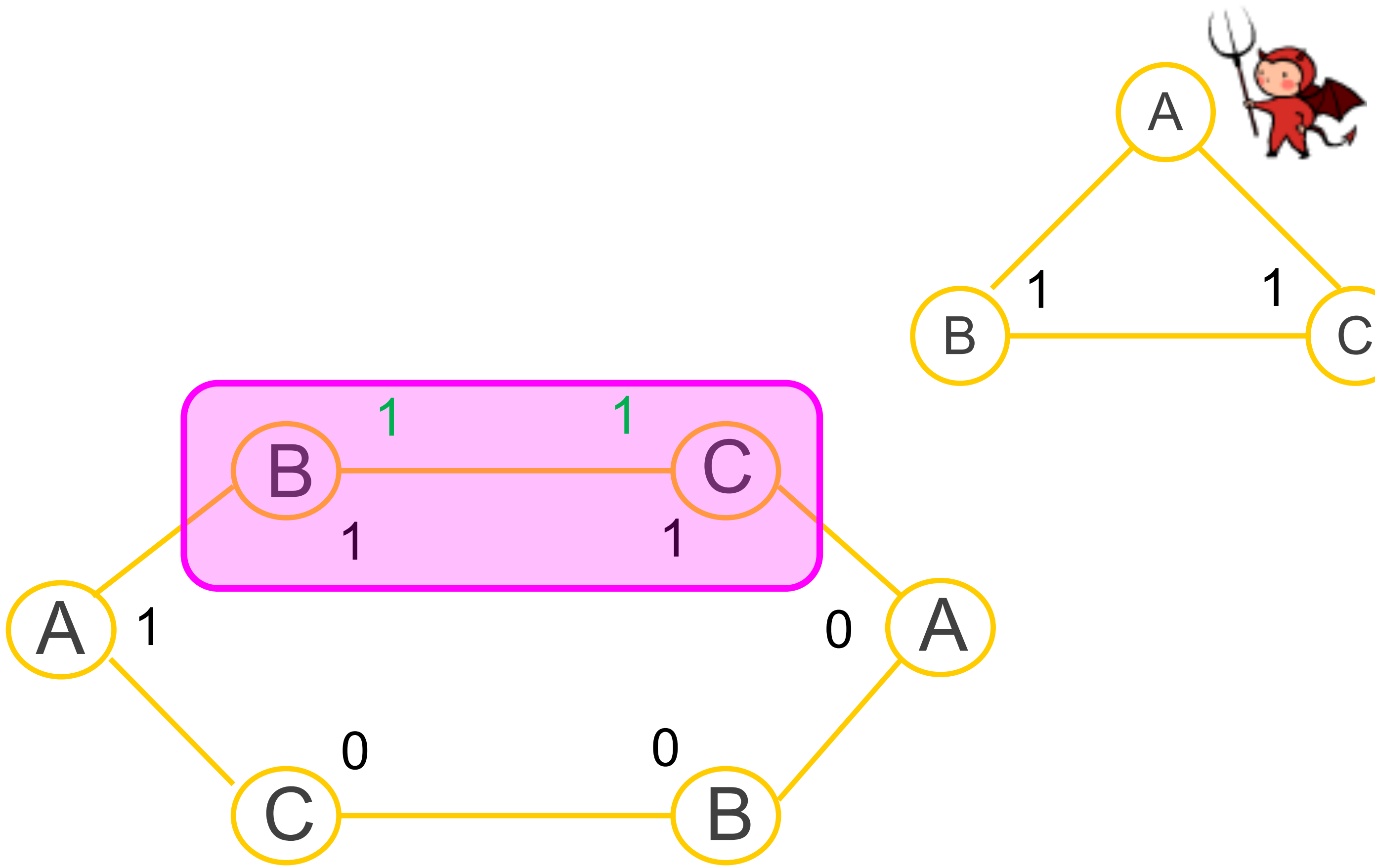


Fischer-Lynch-Merritt Proof

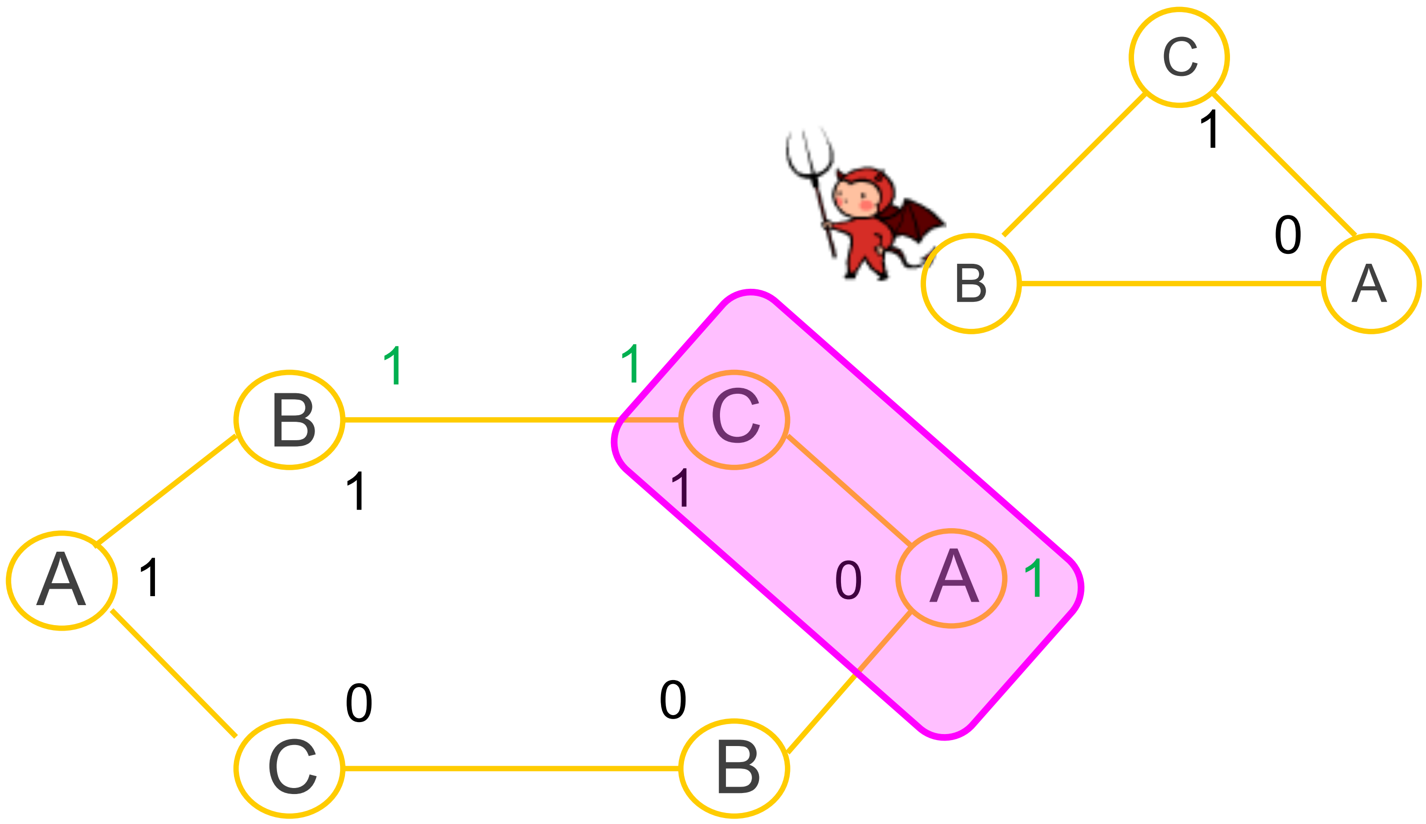
- Connect six non-faulty processes in a ring, let them run the algorithm, and feed them inputs as in the figure



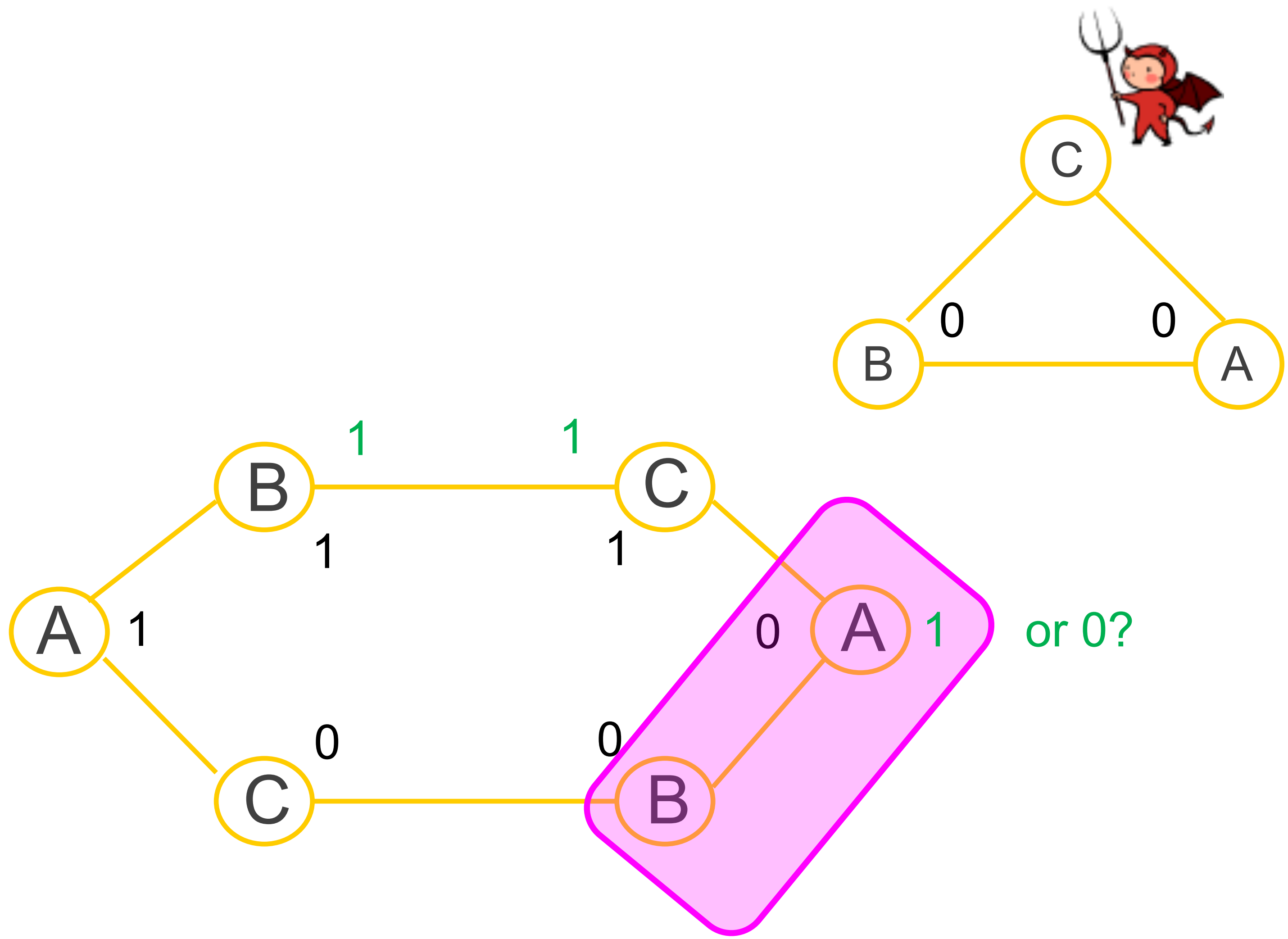
Fischer-Lynch-Merritt Proof



Fischer-Lynch-Merritt Proof



Fischer-Lynch-Merritt Proof



Fischer-Lynch-Merritt Proof

- No algorithm solves BA with $n = 3$, $f = 1$
- Now generalize to any $n \leq 3f$
- Suppose for contradiction that a magic algo solves BA for some n and f where $n \leq 3f$
- We can use it to solve 1-fault-out-of-3 BA

Fischer-Lynch-Merritt Proof

- Use f-out-of-n BA algo to solve 1-out-of-3 BA
  - Each of the three parties simulates ≤ f parties so that the total number of parties is n
    - 1 fault out of 3 → ≤ f faults out of n
  - Run magic algo, 1-fault-out-of-3 BA solved
  - Contradiction, QED
- Where does the proof break down if using signatures?

Fault Bounds So Far

- Crash broadcast and agreement: $f < n$
- Byzantine broadcast (BB) with signatures: $f < n$
- Byzantine agreement (BA): $f < n/2$
- BA or BB without signatures: $f < n/3$

- Now moving on to more practical problems

Broadcast to Replication

- Broadcast gives replication
- Idea: Parties take turns to broadcast values
  - Crashed broadcaster → possibly ⊥ in that position
  - Byzantine broadcaster → possibly invalid value
  - Everyone agrees on those, can simply discard
- This achieves Total-Order Broadcast

Total-Order (Atomic) Broadcast

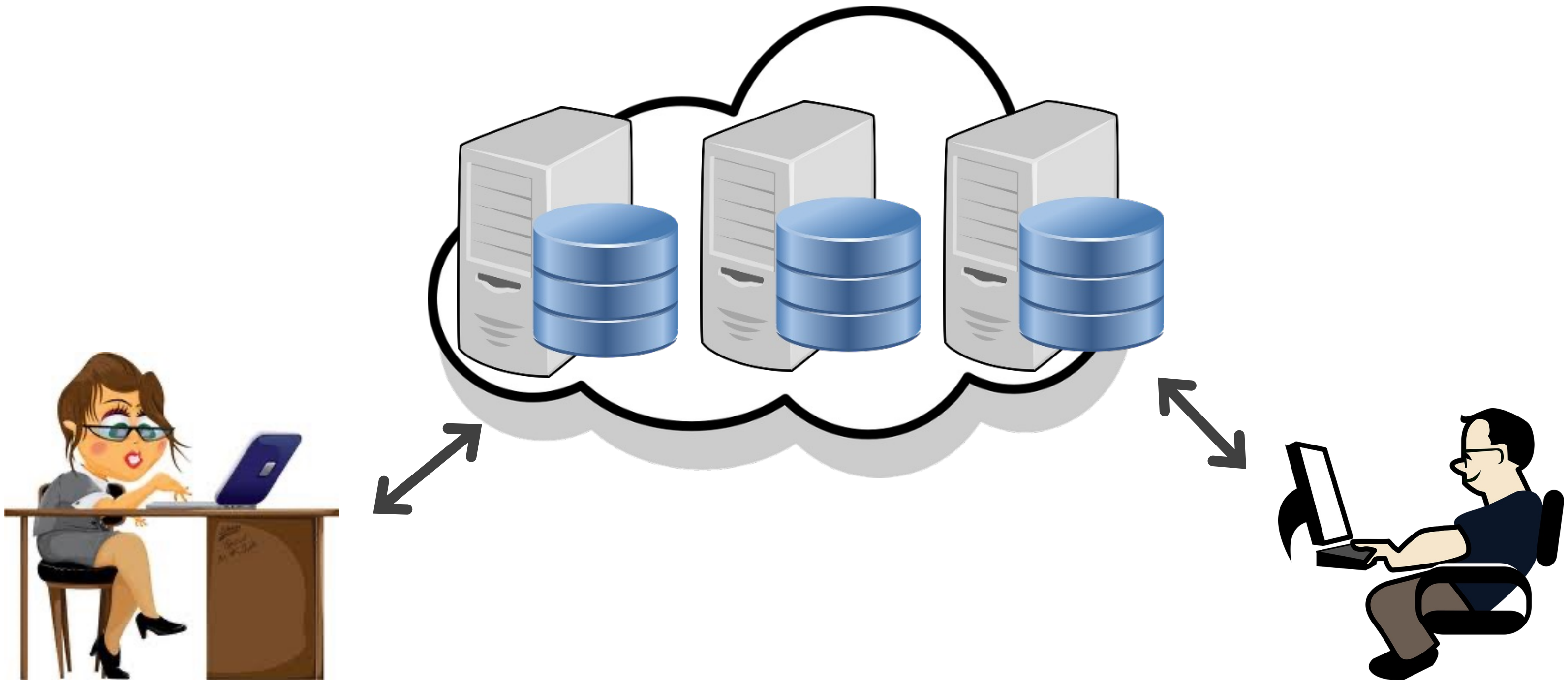
- Parties propose values, and agree on a sequence of values
- Safety: no different values at every position in the sequence
- Liveness: every proposed value eventually added to the sequence
- Validity not needed (no trivial solution)

TO Broadcast vs. Replication

- TO broadcast: parties propose values, and agree on a sequence of values
  - Very close to replication, one subtlety remains
- Replication needs to serve external clients, not just reach consensus among servers
  - Clients do not see inner-working of the protocol

Replication

- External clients propose values (to servers) and external clients agree on a sequence of values

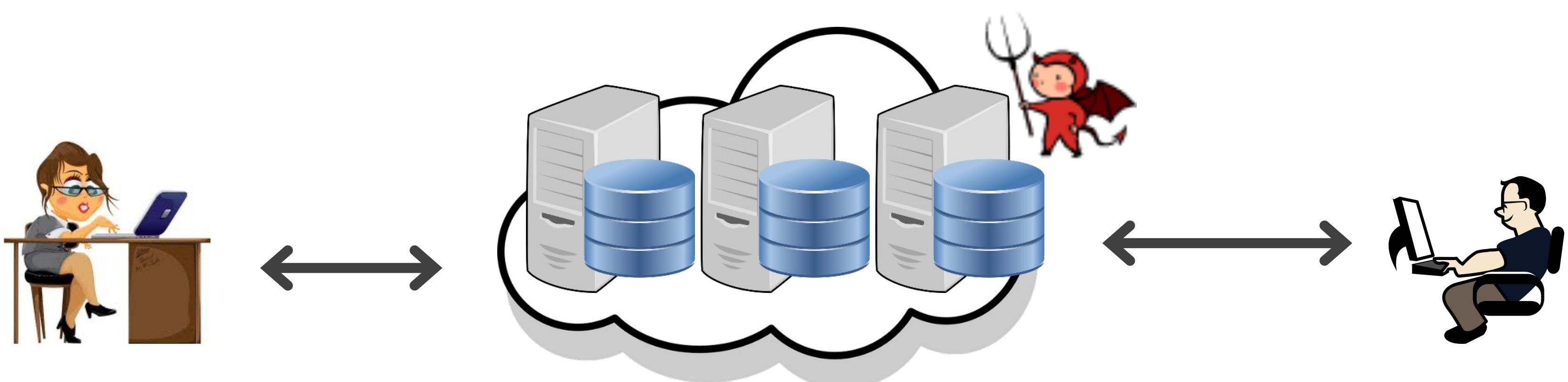


Replication

- External clients propose values (to servers) and external clients agree on a sequence of values
- Safety: no different values at every position in the sequence
- Liveness: every proposed value eventually added to the sequence
- Validity: external (application level)

Replication

- Clients send values to servers; servers run a total-order broadcast and reply to clients
  - Problem solved for crash faults
  - Byzantine server can send a fake reply
    - Solution: require same reply from f+1 servers



Replication Fault Bound

- Byzantine fault tolerant replication requires same reply from f+1 replicas
- Need $n > 2f$ so that honest > Byzantine
- Byzantine replication impossible if $f \geq n/2$
  - Two groups $|P| \leq f$ and $|Q| \leq f$ present different views
  - Client don't know who to believe
    - Cannot distinguish the f Byzantine servers from the (up to) f honest servers

Fault Bounds for Synchrony

- Crash: $f < n$ ~~(ignore agreement)~~
- Byzantine without signatures: $f < n/3$
- Byzantine with signatures:
  - Broadcast and total-order broadcast: $f < n$
  - Agreement and replication: $f < n/2$
- Moving on to asynchrony

Recall Asynchrony

- Any message can take arbitrarily long
  - but will eventually arrive
  - (Asynchrony also says any local computation can be arbitrarily long. But can be lumped into msg delay.)

- Helpful to think of asynchrony as an *adversarial network scheduler*

Broadcast in Asynchrony

- Cannot tolerate a single crash (broadcaster)
  - Same proof as in async impossibility of synchronizer
  - No msg from broadcaster, what do we do?
  - Wait forever? Violate liveness.
  - Move on? Violate validity.

FLP Impossibility

- Under asynchrony, no <u>deterministic</u> agreement protocol can tolerate a single crash fault [Fischer-Lynch-Patterson, 1985]

- Recall configuration and valency
- Step 1: there exists an initial bivalent config
- Step 2: can always stay bivalent

Recall Configurations

- Union of the states of all parties
- A protocol execution is an evolution of configurations: $C_0 \rightarrow C_1 \rightarrow C_2 \ldots$
- In synchrony, evolve after each round
- In asynchrony, evolve after each msg arrival
  - "Msg m arrives at party p" is called an "event"

More on Async Configurations

- $C_0 \rightarrow_e C_1 \rightarrow_{e'} C_2$
- Apply events in what order? Does it matter?
- Must apply e before e' if e happens before e'
  - Type 1: two events with the same recipients
  - Type 2: one event "triggers" another
- Otherwise, apply in either order, same outcome
  - $C \rightarrow_e C_1 \rightarrow_{e'} C_2$ $\qquad$ $C \rightarrow_{e'} C'_1 \rightarrow_e C_2$

Recall Valency

- A config C is **0-valent**, if in all configs reachable from C, honest parties decide 0
  - No matter what happens from now on, decide 0
- A config C is **1-valent,** if ……, all decide 1
- **Univalent** = 0-valent or 1-valent
- **Bivalent** = not univalent

FLP Impossibility Proof

- Step 1: there exists an initial bivalent config
  - Proved in round lower bound

- Step 2: can always stay bivalent
  - What do we have to prove exactly?
  - $\forall$ bivalent C, $\exists$ bivalent C' such that C $\rightarrow$ C' ?

A Warm-Up (Not Actual Proof)

- $\forall$ bivalent C, $\exists$ bivalent C' such that $C \rightarrow C'$
  - Suppose for contradiction all evolution of C univalent
  - $\exists\ e_0, e_1$ s.t. $C \rightarrow_{e_0} C_0$ (0-val) and $C \rightarrow_{e_1} C_1$ (1-val)
  - If $e_0 \parallel e_1$, then $C \rightarrow_{e_0} C_0 \rightarrow_{e_1} C^* == C \rightarrow_{e_1} C_1 \rightarrow_{e_0} C^*$
    - C^* cannot be both 0-val and 1-val, contradiction
  - e_0 and e_1 could not have triggered one another if they both already exist (applicable to C)
  - e_0 and e_1 must have the same recipient p

A Warm-Up (Not Actual Proof)

- $\forall$ bivalent C, $\exists$ bivalent C' such that C $\rightarrow$ C'
  - Suppose for contradiction all evolution of C univalent
  - $\exists e_0$, e_1 with the same recipient p such that

    $C \rightarrow_{e_0} C_0$ (0-val) and $C \rightarrow_{e_1} C_1$ (1-val)
  - Fate of system depends on which msg reaches p first
    - Must wait for p to tell us. What if p does not speak?
    - Can't wait forever; Any decision could be wrong
  - Contradiction. C must have a bivalent evolution

FLP Impossibility Proof

- Step 1: there exists an initial bivalent config
- Step 2: can always stay bivalent
  - What do we have to prove exactly?
  - $\forall$ bivalent C, $\exists$ bivalent C' such that C $\rightarrow$ C' ?
    - Insufficient: may be delaying some events forever
- Actual Step 2: $\forall$ bivalent C, $\forall$e applicable to C, $\exists$ bivalent C' such that C $\rightarrow$ ...... $\rightarrow_e$ C' !
  - All msgs eventually delivered, still bivalent!

FLP Impossibility Proof

- $\forall$ bivalent C, $\forall$ e applicable to C, $\exists$ bivalent C' such that $C \rightarrow \ldots\ldots \rightarrow_e C'$
  - $\mathcal{S}$: set of configs reachable from C w/o applying e
  - $\mathcal{T}$: set of configs by applying e to $\mathcal{S}$
  - Want to prove $\mathcal{T}$ contains a bivalent config
- Proof:
  - Suppose for contradiction all configs in $\mathcal{T}$ univalent
  - Can find S_0 and $S_1 \in \mathcal{S}$ s.t. $S_i \rightarrow_e$ is i-valent
    - Find 0-val A_0 reachable from C. If $A_0 \in \mathcal{S}$, done; Else, trace back to the config before applying e

FLP Impossibility Proof

- ∀ bivalent C, ∀ e applicable to C, ∃ bivalent C' such that C → …… $\rightarrow_e$ C'
  - $\mathcal{S}$: set of configs reachable from C w/o applying e
  - $\mathcal{T}$: set of configs by applying e to $\mathcal{S}$
  - Suppose for contradiction all configs in $\mathcal{T}$ univalent
  - Can find S_0 and $S_1 \in \mathcal{S}$ s.t. $S_i \rightarrow_e$ is i-valent
  - Can find *neighboring* S_0' and $S_1' \in \mathcal{S}$ s.t. $S_0' \rightarrow_{e'} S_1'$ and $S_i' \rightarrow_e$ is i-valent
    - $\mathcal{S}$ is connected, such neighbors must exist
  - $S_0' \rightarrow_e$ is 0-valent, $S_0' \rightarrow_{e'} S_1' \rightarrow_e$ is 1-valent
  - Rest of the proof same as warm-up

FLP Impossibility Proof

– $S_0' \rightarrow_e$ is 0-valent, $S_0' \rightarrow_{e'} S_1' \rightarrow_e S^*$ is 1-valent

Rest same as warm-up:

– e $\nparallel$ e', otherwise S* is both 0–val and 1-val

– So e and e' have the same recipient p

– Fate depends on which msg arrives at p first

– What if we don't hear from p?

– Can't tell if p crashed or is just slow

– Can't wait forever; Any decision could be wrong

FLP Impossibility

- FLP does not say asynchronous consensus is impossible! Randomized consensus possible.
- Where does the proof rely on "deterministic"?
- Does it mean every deterministic protocol ALWAYS fails under asynchrony?
  - No, just says it *can* fail, can also get lucky.

What can we do given FLP?

- Consider easier problems
- Randomization
  - asynchronous agreement, total order broadcast, and replication possible under randomization
  - Single-value broadcast still impossible
- Consider easier models (partial synchrony)
  - Single-value broadcast still impossible under psync

Partial Synchrony

- (Intuitively) The network is sometimes asynchronous and sometimes synchronous
  - Maintain safety during asynchronous periods
  - Achieve liveness during synchronous periods

Partial Synchrony

- (Formally) There exists an **unknown** Global Standardization Time (GST) after which the network becomes synchronous
  – Forever synchronous after GST???
    - Hope to capture “sufficiently long sync periods”
  – Unknown to whom?
    - Can be viewed as a game between protocol designer and the adversary

Psync Agreement Fault Bound

- Crash: $f < n/2$
  - Proof: Two groups $|P| \le f$ and $|Q| \le f$
  - Scenario I:
  - Scenario II:
  - Scenario III:

Psync Agreement Fault Bound

- Crash: $f < n/2$
  - Proof: Two groups $|P| \leq f$ and $|Q| \leq f$
  - Scenario I: P non-faulty & receive v, Q crash
    - P eventually commit v due to validity
  - Scenario II: Q non-faulty & receive v', P crash
    - Q eventually commit v' due to validity
  - Scenario III: Both non-faulty, P receive v, Q receive v' GST sufficiently large → Both think the other crashed
    - P commit v, Q commit v'

Psync Agreement Fault Bound

- Byzantine: $f < n/3$
  - Proof: Three groups $|P| \leq f$, $|Q| \leq f$, $|R| \leq f$
  - Scenario I:
  - Scenario II:
  - Scenario III:

Psync Agreement Fault Bound

- Byzantine: $f < n/3$
  - Proof: Three groups $|P| \leq f$, $|Q| \leq f$, $|R| \leq f$
  - Scenario I: P/R non-faulty & receive v, Q crash
    - P eventually commit v due to validity
  - Scenario II: Q/R non-faulty & receive v’, P crash
    - Q eventually commit v’ due to validity
  - Scenario III: P non-faulty & receive v, Q non-faulty & receive v’, R Byzantine behave towards P like in I and towards Q like in II. GST sufficiently large.
    - P cannot distinguish from I, commit v
    - Q cannot distinguish from II, commit v’

Async and Psync Fault Bounds

- Agreement under partial synchrony
  - Crash: $f < n/2$
  - Byzantine: $f < n/3$ (nothing to do with signatures)
- Both bounds apply to async or randomized
- Both bounds apply to TO-bcast and replication
  - Standard (single-value) broadcast still cannot tolerant even a single crash!

Fault Bounds Summary

- Async deterministic: $f = 0$
  - Broadcast, agreement, total-order bcast, replication
- Psync or randomized async
  - Broadcast: $f = 0$
  - Agreement, total-order broadcast, or replication: crash: $f < n/2$, Byzantine: $f < n/3$
- Sync
  - Crash: $f < n$ for all four problems
  - Byzantine no signature: $f < n/3$ for all four problems
  - Byzantine with signature
    - $f < n$ for broadcast and total-order broadcast
    - $f < n/2$ for agreement and replication

Fault Bounds Better Summary

- Byzantine agreement: $f < n/2$
- Byzantine replication: $f < n/2$
- Byzantine no signature: $f < n/3$
- Async deterministic: $f = 0$
- Psync broadcast: $f = 0$
- Psync crash: $f < n/2$
- Psync Byzantine: $f < n/3$