ILLINOIS
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Lecture 11: Weaker Broadcast & Agreement in Asynchrony

CS 539 / ECE 526

Distributed Algorithms

Impossibilities of Fault Tolerance in Asynchrony

- Under asynchrony, no broadcast protocol can tolerate a single crash fault (sender)

- Under asynchrony, no deterministic agreement protocol can tolerate a single crash fault
  - Fischer-Lynch-Paterson, 1985

What can we do?

- Consider easier problems
- Randomization
- Consider easier models (partial synchrony)

- Agreement, total order bcast, and replication possible in psync or async with randomization
  - Single-value broadcast still impossible

Outline

- Consider easier problems in asynchrony
  - Reliable and consistent broadcast
  - Graded agreement

Relaxing the Broadcast Problem

- n parties, including a designated sender with an input x, up to f faulty

- Safety: no different outputs
- Liveness: everyone outputs
- Validity: sender honest → everyone outputs x

- Cannot ask for both “liveness under faulty leader” and “validity under honest leader”
- Will relax liveness under faulty leader

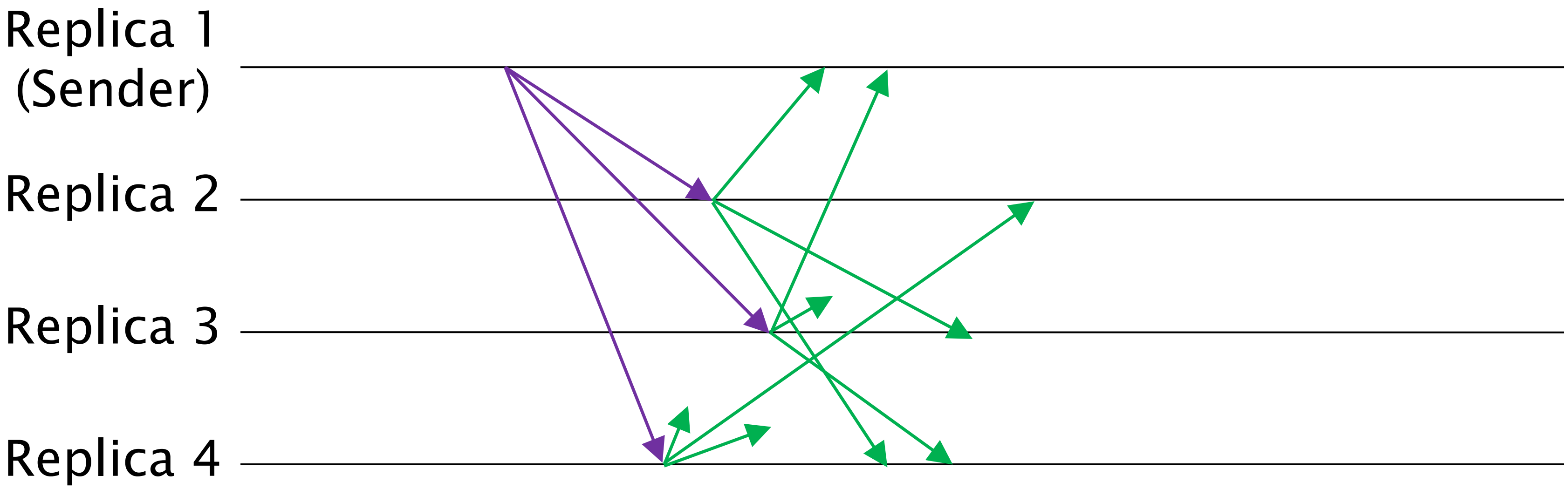
Reliable Broadcast (RBC)

- n parties, including a designated sender with an input x, up to f faulty

- Safety: no different outputs
- **Liveness**: **either everyone outputs or no one outputs**
- Validity: sender honest → everyone outputs x

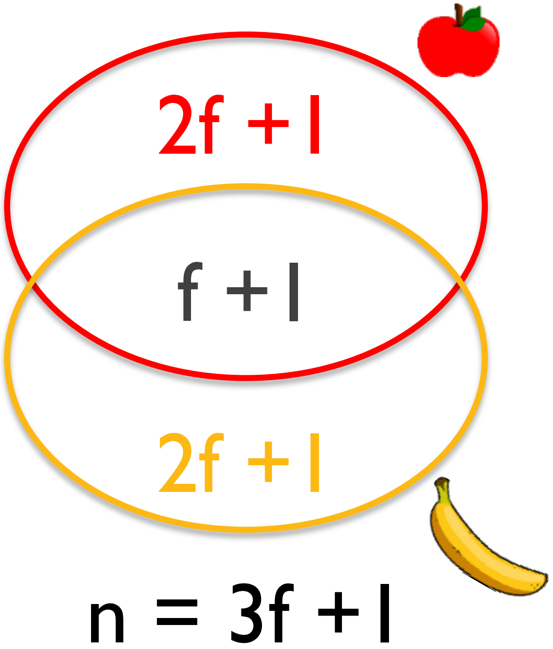
A Simple Byzantine RBC

- f < n/3, use signatures
- Sender proposes x; replicas send signed votes
- Upon receiving n-f votes for x, output x, and forward these votes to all other replicas



Safety: Quorum Intersection

- Some honest outputs v → 2f+1 votes for v → f+1 honest votes for v → at most 2f votes for v’ → no honest outputs v’



2f +1

f +1

2f +1

n = 3f +1

Liveness and Validity

- Validity: an honest sender proposes v to all → all honest eventually vote v → all output v

- Liveness: an honest outputs → it forwards a quorum of votes to all honest → all output
  - Hence, either all output or no one outputs
  - A quorum of votes is a *transferrable certificate*

- How does a malicious sender prevent liveness?

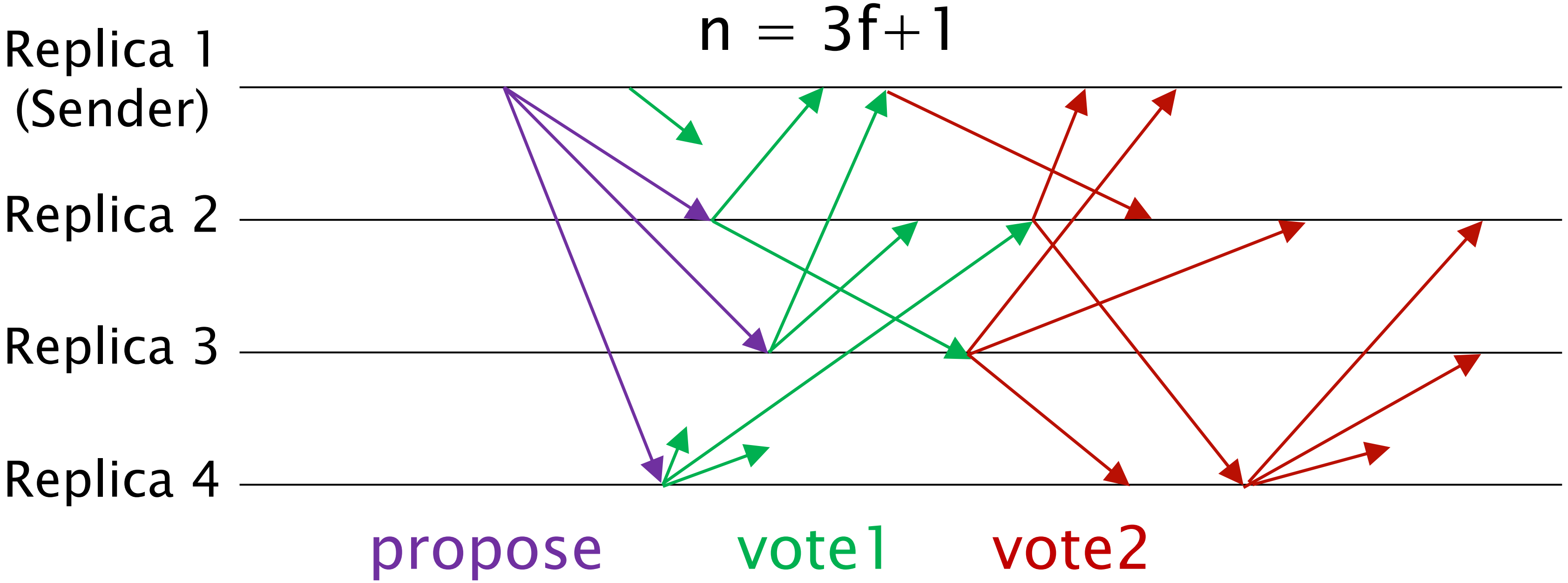
Byzantine RBC Efficiency

- Round complexity:
  - Under good leader: commit in 2, terminate in 3

- Communication complexity:
  - $O(n^2)$ messages
  - $O(n^3|\sigma|)$ bits

Bracha's Byzantine RBC

- Leader proposes x; replicas send vote1
- Upon receiving n-f matching vote1, send vote2
- Upon receiving f+1 matching vote2, send vote2
- Upon receiving n-f matching vote2, output



n = 3f+1

Replica 1 (Sender)

Replica 2

Replica 3

Replica 4

propose

vote1

vote2

Bracha RBC Correctness

- Safety: quorum intersection

- Validity: an honest sender proposes v to all → all vote1 → all vote2 → all output

- Liveness: an honest outputs → n-f vote2 → n-2f = f+1 vote2 from honest → all vote2 → all output
  - An "amplification" of vote2

Bracha RBC Efficiency

- Round complexity:
  - 3 or 4 rounds

- Communication complexity:
  - O(n^2) msgs
  - O(n^2) bits
  - Signature-free

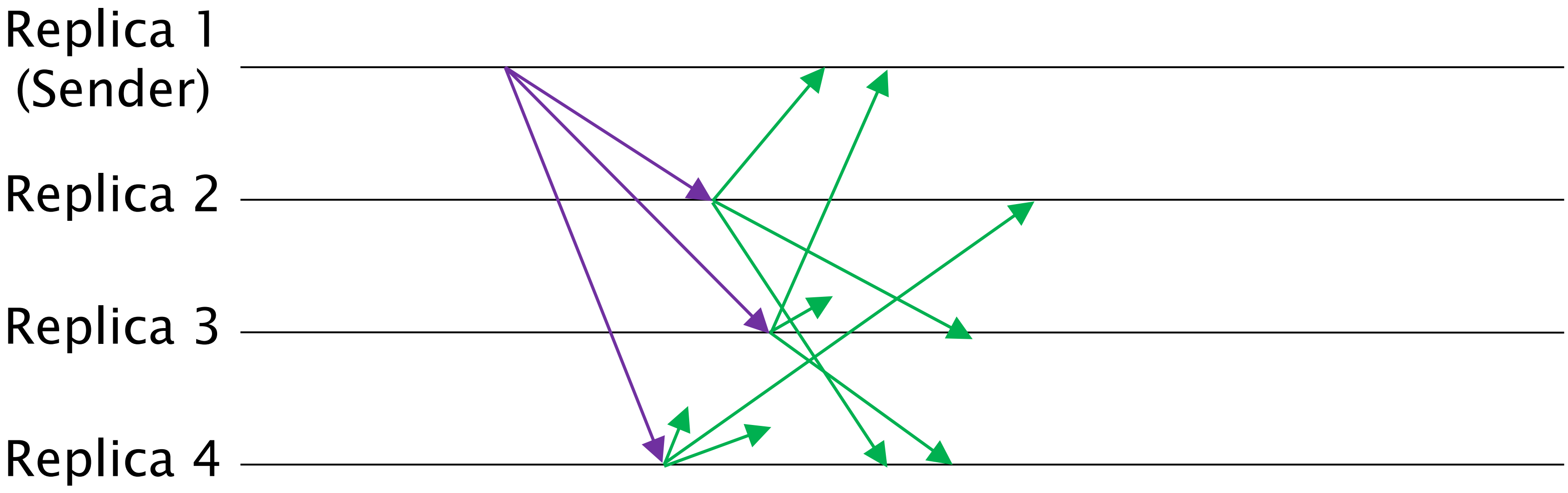
Consistent Broadcast (CBC)

- n parties, including a designated sender with an input x, up to f faulty

- Safety: no different outputs
- **Liveness**: **none**
- Validity: sender honest → everyone outputs x

A Simple Byzantine CBC

- $f < n/3$
- Sender proposes x; replicas send votes
- Upon receiving n-f votes for x, output x



Correctness and Efficiency

- Safety: quorum intersection
- Validity: an honest sender proposes v to all → all vote → all output

- 2 rounds
- $O(n^2)$ messages (all-to-all voting)

Outline

- Consider easier problems in asynchrony
  - Reliable and consistent broadcast
  - Graded agreement

Graded Agreement (GA)

- n parties, each with an input, up to f faulty
- Each party outputs value y and “grade” bit g
  - g is roughly “confidence”

- Liveness: everyone outputs
- Validity: every non-faulty inputs x → every non-faulty outputs (x, 1)
- Safety: no distinct confident outputs: no two non-faulty output (y, 1) and (y’, 1) with y ≠ y’
  - Other variants exist

Async GA for f < n/2 Crash

- Party j has input x_j:
  - Round 1: party j sends (vote, x_j)
    - Wait for n-f = f+1 vote msgs (n=2f+1)
  - If all f+1 votes are for the same x, then output (x, 1); Else, output (x', 0) for any x' with one vote
    - Will just output own input

GA Correctness

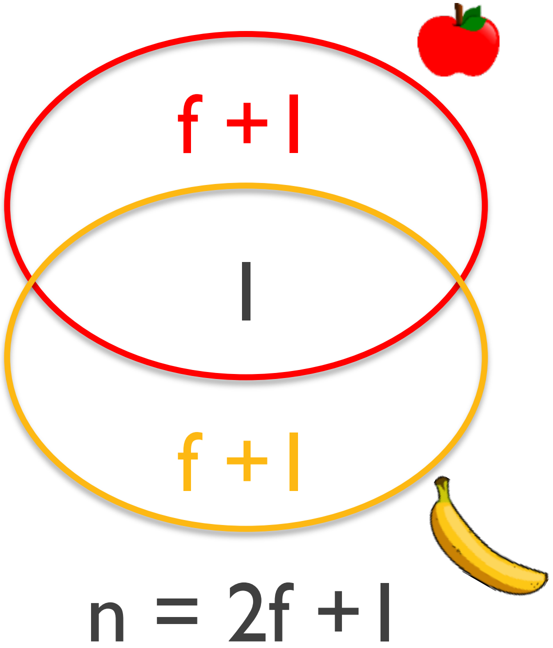
- Liveness: waits for n-f msgs, will get that many

- Validity: same input x → matching votes → everyone outputs (x, 1)

- Safety: quorum intersection

Quorum Intersection (Crash)

- Impossible to have two non-faulty party output (x,1) and (x’,1) for x’ ≠ x



Graded Agreement (GA)

- n parties, each with an input, up to f faulty
- Each party outputs value y and “grade” bit g
  - g is roughly “confidence”

- Liveness and validity as before
- Many variants of safety:
  - S1: No $(y, 1)$ and $(y', 1)$ for $y \neq y$
  - S2: One outputs $(y, 1)$, all output $(y, *)$
  - S3: No $(y, *)$ and $(y', *)$ for $y \neq y'$, $y \neq \perp$, $y' \neq \perp$

GA Safety Variant Relations

- S1: No $(y, 1)$ and $(y', 1)$ for $y \neq y$
- S2: One outputs $(y, 1)$, all output $(y, *)$
- S3: No $(y, *)$ and $(y', *)$ for $y \neq y'$, $y \neq \perp$, $y' \neq \perp$

- S2 strictly stronger than S1
- S3 strictly stronger than S1
  - With a reasonable assumption that $\perp$ cannot be output with confidence
- S3 does not imply S2: $(y, 1)$ and $(\perp, 1)$
- S2 does not imply S3: $(y, 0)$ and $(y', 0)$

Async GA for f < n/2 Crash

- Party j has input x_j:
  - Round 1: party j sends (vote1, x_j)
    - Wait for n-f = f+1 vote1 msgs (n=2f+1)
  - Round 2: if all f+1 vote1 are for the same x, party j sends (vote2, x); else, sends (vote2, ⊥)
    - Wait for n-f = f+1 vote2 msgs (n=2f+1)
  - If all f+1 vote2 are for the same x, then output (x, 1); Else if there is one vote2 for x, then output (x, 0); Else, output (⊥, 0).

GA Correctness

- Liveness: waits for n-f msgs, will get that many

- Validity: same input x → matching vote1 → matching vote2 → → everyone outputs (x, 1)

- Safety: quorum intersection → at most one non-⊥ value in vote2 → both S2 and S3

Summary

- Broadcast (the strongest formulation) is impossible with a single crash under psync

- Weaker primitives are possible in async:
  - Reliable or consistent broadcast
  - Graded agreement
  - May even be useful in sync

- Quorum intersection & certificates are common tools in psync / async

Graded Broadcast (Gradecast)

- n parties, including a designated sender with an input x, up to f faulty
- Each party outputs value y and "grade" bit g
  - g is roughly "confidence"

- Liveness: everyone outputs
- Validity: every non-faulty inputs x → every non-faulty outputs (x, 1)
- Safety: many variants similar to GA
- Impossible in psync/async but useful in sync